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INVESTIGATION OF THE NATURAL VIBRATIONS OF MACHINE ELEMENTS USING THE HILBERT TRANSFORM

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An experimental method for investigation of the elastodissipative properties of nonlinear oscillatory systems is based on use of the instantaneous frequency and instantaneous amplitude of the natural vibrations. An example of analysis of a nonlinear system is included.

The characteristics of dynamic systems can be determined by analysis of experimentally obtained records of free (damped) vibrations [1-3]. Only a small number of peak (amplitude) value of the damping process, spread out in time over the period or half-period of the vibrations, are used to estimate, for example, energy-dissipation characteristics. Naturally, these small samples cannot be used to determine the dissipation characteristics with high accuracy, especially when the latter depend nonlinearly on the natural-vibration amplitude.

In this paper we propose a method for analysis of natural vibrations based on use of the instantaneous amplitude (envelope) and instantaneous frequency of a damping narrow-band process for use in experimental investigation of the elastodissipative properties of quasi-linear oscillatory systems with one degree of freedom in response to a pulse disturbance:

$$m\ddot{x}+H(\dot{x})+K(x)=0, \tag{1}$$

where m is the mass and $H(\dot{x})$ and $K(x)$ are symmetric dissipative and elastic forces. It will be recalled that according to analytic-signal theory [4], any process, including narrow-band natural vibrations of system (1), can be represented in the form of the following combination of slowly varying functions: $x(t)=A(t)\cos\psi(t)$, where $A(t)$ is the instantaneous amplitude and $\dot{\psi}(t)=\omega(t)$ the instantaneous circular frequency of the natural vibrations.

To obtain the instantaneous amplitude $A(t)$ and instantaneous frequency $\omega(t)$ from the original process $x(t)$, it is necessary to apply the integral Hilbert transformation [4]

$$A(t)=\sqrt{x^2(t)+x_H^2(t)},\quad \omega(t)=[x(t)\dot{x}_H(t)-x_H(t)\dot{x}(t)]/A^2(t),$$

where $x_H(t)=-\pi^{-1}\int_{-\infty}^{\infty}\frac{x(\tau)}{(\tau-t)}d\tau$ is the Hilbert conjugate of the original natural-vibration process $x(t)$. In natural vibration of a system that admits of a unique quasi-harmonic regime, the rate of decrease of the instantaneous amplitude in time is determined by the form of the symmetric dissipative function $H(\dot{x})$, and the variation of the instantaneous frequency in time by the symmetric elastic restoring forces $K(x)$.

In the particular case of free vibrations of a linear system $m\ddot{x}+2mh\dot{x}+kx=0$ the instantaneous frequency, which is practically equal to $\sqrt{k/m}$, does not vary in time, while the instantaneous amplitude decreases exponentially with a constant damping coefficient h. In the general case of investigation of nonlinear systems, the instantaneous damping coefficient $h(t)$ is a function of time:

(2)

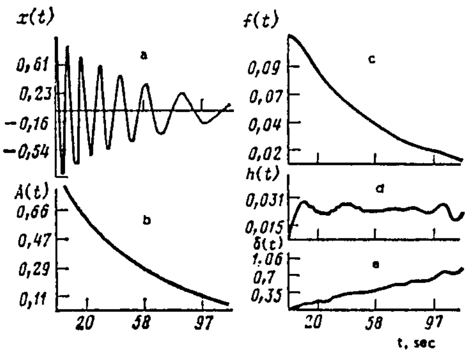

Fig. 1

where A_0 is the initial amplitude at t = 0.

In practice, the influence of the dissipative forces is usually taken into account with a dimensionless characteristic - the logarithmic vibration decrement δ, which is constant for a linear system [3], $\delta = 2\pi h/\omega$, where ω is the circular frequency of the natural vibrations. Applying (2), we have for the instantaneous logarithmic vibration decrement

$$\delta(t) = f^{-1}(t)\, t^{-1} \ln [A_0/A(t)], \tag{3}$$

where $f(t) = \omega(t)/2\pi$ is the instantaneous frequency of the free vibrations.

It follows from expressions (2) and (3) that the instantaneous damping coefficient and the vibration decrement are function of time and can be determined at any point of the damped process. Since the total number of points that map the free vibration discretely is much larger than the number of peak points, avenues are open for application of statistical methods to processing of the experimental information, which improve the estimates of these characteristics, in subsequent determination of the elastodissipative characteristics of the oscillatory systems.

Let us consider typical cases of interaction between the natural-vibration characteristics of system (1). If the system to be tested has nonlinearized elastic forces, the natural-vibration frequency will be decisively related in most cases to the amplitude of the vibrations. Neglecting the insignificant effect of the dissipative forces on the vibration frequency, we represent this departure from synchronism of the natural vibrations in the form of a regression curve of the instantaneous amplitude on the instantaneous frequency, which represents a kind of skeleton curve of the system being tested. Therefore the proposed method for analysis of the damped process offers a way to plot the skeleton curve of the quasi-linear system (1) directly. Subsequent analysis of the topography of the skeleton curve is essential in evaluating the properties of the particular vibrating system, e.g., in reconstructing the characteristics of the elastic forces [5].

If nonlinear dissipative forces are operating in the test system, the values obtained from the instantaneous vibration decrement may depend on the instantaneous amplitude and (or) on the instantaneous frequency of the vibrations. In tests of dynamic systems, it would naturally be desirable to process the test data in such a way as to reflect the trend of the decrement as a function of amplitude and frequency as fully and accurately as possible in the form of a function of two independent variables. As we indicated, however, the natural-vibration frequency is, in the general case, related to the amplitude, so that it is not possible in experimental analysis of the natural vibrations of system (1) to tell whether the friction forces are dependent on frequency or not. One possible exception is found in the simplest types of linear damping in the presence of nonlinear elasticity, when it is possible on the basis of the presence or absence of an effect of frequency on the vibration decrement to infer that friction is viscous or internal. Experience from experimental studies of the vibration of engineering structures indicates that the nature of the dissipative forces is such that the frequency has practically no influence on the value of the logarithmic decrement, and that a model of frequency-independent (internal) friction should be used to describe the vibrations in these cases [3].

As a model example, let us consider the natural vibrations and instantaneous characteristics of the process in the elementary nonlinear system $\ddot{x}+2\cdot 0{,}025\dot{x}+x^3=0$ (Fig. 1a), which were determined on a computer for 300 points at 0.8-sec intervals $(x_0=1, \dot{x}_0=0)$. Use of digital-

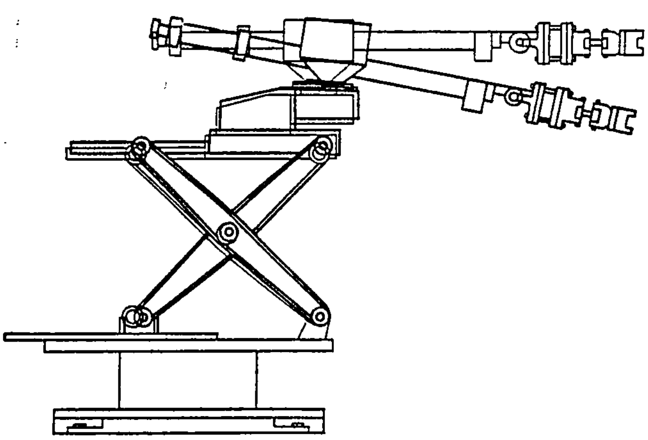

Fig. 2

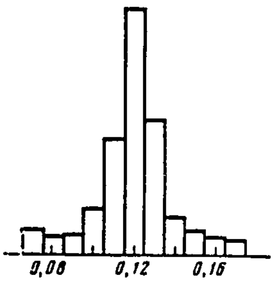

Fig. 3

filtration programs [6] for the Hilbert transformation with less than 1% nonuniformity of the frequency characteristics ensured high accuracy in calculating from the initial process (Fig. 1a) the instantaneous amplitude (Fig. 1b), the instantaneous frequency (Fig. 1c), the instantaneous damping decrement (Fig. 1d), and the instantaneous vibration decrement (Fig. 1e). The straight line obtained for the regression of the instantaneous circular frequency on the instantaneous amplitude $\omega(A) = 0.84$ A practically coincides with the theoretical skeleton curve of the test system $\omega(A) = 0.8472$ A [7]. The inversely proportional dependence of the instantaneous vibration decrement (Fig. 1e) on the instantaneous frequency (Fig. 1c) and the practically constant value of the damping decrement $h = 0.024$ (Fig. 1d) point to the conclusion that viscous linear damping is operative in the system in this case, and that the reconstructed value of the damping coefficient differs from theory by only 4%.

To illustrate, let us consider an experimental study of free vibrations in the elastic system of a Universal-5 manipulator robot. When the robot was tested under realistic service conditions, pulsed vibrations were observed simultaneously in several natural-vibration modes with different frequencies; we therefore put the signal through preliminary band-pass filtration to study the properties of the vibrational system in isolation in the neighborhood, for example, of the 11.3 Hz natural frequency at which the pantograph is shifted relative to the platform and the arm is turned relative to the body (Fig. 2). The 4096 points of the process at a discretization frequency of 160 Hz that were processed with the computer enabled us to obtain the corresponding instantaneous characteristics of the damping vibrations and, for example, the trivial skeleton curve of a practically linear system. The scatter of the instantaneous vibration decrement was insignificant (Fig. 3), and its mean value equal to 0.12.

The skeleton curves and energy-dissipation characteristics of systems as complex as robots had hitherto been obtained through highly time-consuming resonance tests of the structure with variation of the forced-vibration amplitude. The proposed method reduces the time required for tests to determine these characteristics to a small fraction by virtue of the separation of the instantaneous amplitude and frequency of the damped process, which can be measured with pulsed disturbances in a time comparable to the system's natural-vibration frequency. Analysis of the dynamic characteristics obtained for the robot points to the conclusion that the vibration mode of the structure at 11.3 Hz is the second natural vibration mode, which had appeared under actual service conditions without artificial excitation of the system. The nonvarying natural frequencies and vibration decrement found here validate, for example, the use of a linear vibrational-system model that describes the periodic shifting and rotation of the pantograph relative to the platform in the horizontal plane, as well as the displacement of the arm in its housing in both planes.

Thus, by virtue of the fact that the dissipative and nonlinear elastic forces have totally different effects on the natural vibrations (energy dissipation lowers the instantaneous amplitudes, while nonlinear elasticity links the instantaneous frequency and amplitude in a certain relationship), it becomes possible to determine aspects of the behavior of these forces. For this identification, we propose that relationships be constructed between the instantaneous frequency and amplitude together with curves of the instantaneous decrement as a function of vibration frequency and amplitude. For acquisition of the regression relationship indicated above in experimental study of free vibrations, we have proposed a new approach based on use of the Hilbert transformation to separate the instantaneous amplitude and frequency and the instantaneous vibration decrement as functions of time from the damped process. It is helpful to use the technique developed here in experimental study of energy-dissipation characteristics in machine designs.

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